

Characterization and Modeling of Underwater Acoustic Communications Channels for Frequency-Shift-Keying Signals

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Abstract – In a fading channel, bit error rate for frequency-shift-keying signals is determined predominantly by the envelope amplitude fading statistics of the signal. The narrowband envelope amplitude distributions are measured from the TREX04 data (as a function of frequency) using M-sequence signals centered at 17 kHz with a 5 kHz bandwidth. The results do not fit the Rayleigh, Rician, Nakagami m -distributions. In contrast, we find that the data are fitted well by a K -distribution. We also analyze the data in terms of long-term and short-term statistics. The long-term and short-term fading statistics are well fitted by the lognormal distribution and Rayleigh distribution respectively, choosing the average time scale to be ~ 0.2 sec. The joint probability distribution function of a lognormal and the Rayleigh distribution is approximately the K -distribution.

I. INTRODUCTION

For underwater acoustic communications, the channel is characterized by (i) a long multipath delay, which extends over many symbols causing inter-symbol interference (ISI), (ii) a high Doppler spread which implies short channel coherence time, and (iii) a time-varying Doppler shift due to the relative platform speed compared with the sound speed.

This paper addresses channel characterization in the frequency domain, specifically the signal envelope amplitude statistics, which forms the basis of bit error rate predictions for MFSK signals. For M-ary frequency-shift-keying (MFSK) signals, the symbols are spread over the frequency band and modulated in both frequency and time. To avoid ISI interference, the symbol duration (including the guard time if appropriate) should be longer than the multipath spread, but in practice this is often not the case. This method is referred to as incoherent communication, since each symbol is detected by an energy detector (for each time-frequency grid). It is less sensitive to the channel temporal fluctuations and does not require a channel equalizer.

The frequency components (bins) in the MFSK signaling are, in theory, orthogonal to each other, implying that there is no leakage of the symbol energy from one frequency channel to the other. In practice, this is not the case due to time-variant nature of the channel. Inter-frequency bin leakage can be substantial if there is significant error in the Doppler shift estimation. To minimize this effect, the frequency bin width Δf is often chosen to be much larger than the uncertainty in the Doppler shift estimation. Channel characterization for MFSK modulation requires estimation of the channel spectrum (the

channel transfer function) as a function of frequency and time. The bit error rate results not only from the noise but also from the ISI and inter- (frequency) channel interference (ICI). For bit error rate modeling/prediction in a realistic channel, the appropriate channel transfer function needs to include the effects of ISI and ICI.

Bit error rate (BER) for MFSK signals depends on the envelope amplitude fading statistics as a function of frequency. Rayleigh and Rician amplitude probability distributions are two commonly assumed models for signal fading in radio frequency (RF) communications [1-3]. For low frequency (e.g., < 1 kHz) sound propagation, Rayleigh and Rician statistics are associated with saturated and partially saturated schemes in which the multipaths are totally random or partially random. A discussion of the statistics for a narrowband signal can be found [4]. We find that neither of the above distributions holds for high frequency underwater acoustic communication signals. We deduce the channel spectrum level fluctuation statistics from data collected at sea, and provide a physics-based interpretation.

Section II describes characteristics of amplitude fluctuations. Narrowband envelope amplitude distribution statistics are deduced from data covering a wide band (4 kHz) of frequencies. Section III reviews candidate fading statistical models. Section IV determines an appropriate model for underwater acoustic channel. Section V provides conclusions.

II. CHARACTERISTICS OF AMPLITUDE FLUCTUATIONS

A. MFSK Modulation

For a narrowband signal in a linear time-variant channel, the channel transfer function can be defined by

$$R(t, f) = H(t, f)S(f), \quad (1)$$

where H is the time-variant channel transfer function at frequency f , R is the received signal and S is the source amplitude. MFSK signals consist of many narrowband signals at frequencies f_k , separated by Δf , where $k = 1, \dots, K$,

$$R(t, f_k) = H(t, f_k)S(t, f_k), \quad (2)$$

where $S(t, f_k)$ is the transmitted symbol sequence in frequency bin f_k at time t , $S(t, f_k) = 0$ or 1 . Each symbol has a time duration $\Delta t = 1/\Delta f$. Detection of symbols at the receiver is based on the symbol intensity $|R(t, f_k)|^2$ which is heavily influenced by the channel spectral level $|H(t, f_k)|^2$. Hence, BER

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modeling/prediction requires knowledge of the statistical distribution of $|H(t, f_k)|$, the envelope amplitude statistics.

Snapshots of the channel transfer functions are estimated from received data using pseudo-random signals, e.g., m-sequences,

$$|H(t_m, f_k)|^2 = |R(t_m, f_k)|^2 / |S(t_m, f_k)|^2, \quad (3)$$

where $|R(t, f_k)|^2$ and $|S(t, f_k)|^2$ are the spectral level of the received and transmitted signals as a function of frequency at $t_m = m \Delta t$, for the m -th symbol. The M-sequence signals, have a flat spectrum when averaged over many sampling periods.

B. Envelope Amplitude Statistics

The M-sequence data were transmitted in consecutive packets, each of 15 sec duration with built in time gaps. Total transmission time was ~ 40 minutes. The M-sequence data are processed first by removing the transducer's frequency response from the received data. The beginning of the M-sequence in each packet is determined by matched filtering the data using either the probe signal before the M-sequence or the first M-sequence, a standard processing technique for communications. The M-sequence data are then Fourier transformed with a window size equal to the symbol duration, (e.g., 1/80 sec). The channel spectrum is obtained using Eq. (3), with the received and transmitted data processed in the same way.

The mean spectral level $\Sigma(f_k)$ of the channel transfer function is estimated by summing the spectrum level over all channel transfer function snapshots and dividing the result by the number of samples. We find that $\Sigma(f_k)$ decreases by as much as 5 dB at the edge of the frequency band. These frequency components are discarded in our analysis. The $\Sigma(f_k)$ varies by 1-2 dB within the 4 kHz bandwidth, which is attributed to the uncertainty in the transducer response curve, which was under-sampled in the original (calibration) measurements. We remove this effect by the following operation:

$$|H'(t_m, f_k)|_{new} = |H(t_m, f_k)| / \sqrt{\Sigma(f_k)}, \quad (4)$$

where t_m denotes the symbol sequence in time. Since the data were transmitted in packets, one has $t_m = n T_0 + j \Delta t$, where n is the packet number, $n = 1, \dots, 134$. T_0 is the time separation between packets, and j is the symbol number within a packet, $j = 1, \dots, 856$. Henceforth we will drop the "prime" and denote the data by $|H(t_m, f_k)|$.

The frequency coherence bandwidth can be measured by cross correlating the channel transfer functions between two different frequencies denoted by its frequency index $k1$ and $k2$,

$$\rho((k2 - k1)\Delta f) = \left\langle \frac{\left| \sum_m H^*(t_m, f_{k1}) H(t_m, f_{k2}) \right|}{\sqrt{\sum_m |H(t_m, f_{k1})|^2} \sqrt{\sum_m |H(t_m, f_{k2})|^2}} \right\rangle, \quad (5)$$

where the correlation is done for each packet (summing over j for a fixed n) and then averaged over all the packets. We find that $\rho < 0.2$ when $k1 \neq k2$, indicating that the channel transfer

functions are uncorrelated between frequency bins. In other words, the frequency coherence bandwidth is < 80 Hz.

For each frequency bin f_k , we determine the probability distribution of the envelope amplitude (or the histogram) $|\tilde{H}(t_m, f_k)|$. The distributions are used to compare with some theoretical fading statistical models.

III. FADING STATISTICAL MODELS

A narrowband signal can be represent by $p(f) = H(f)e^{i2\pi f t}$, where $H(f)$ is the complex amplitude, $H(f) = X + iY$, where X and Y are often referred to as the in-phase and quadrature components of the signal. BER of MFSK signals is determined by the fading statistics of the (envelope) amplitude $Z = \sqrt{X^2 + Y^2}$ as a function of frequency.

A. Models with Gaussian Assumption

Assume that both X and Y are Gaussian random variables with probability distributions given by $X \sim N(\mu_x, \sigma_x)$ and $Y \sim N(\mu_y, \sigma_y)$. The correlation coefficient of two Gaussian random variables is defined as

$$\rho_{xy} = \frac{E(XY) - \mu_x \mu_y}{\sigma_x \sigma_y}. \quad (6)$$

The distribution of the (envelope) amplitude can be expressed in terms of $\mu_x, \mu_y, \sigma_x, \sigma_y, \rho_{xy}$ as given in Eq.(21) of [4] in the context of a propagation model. It can be shown that the (envelope) amplitude distribution so obtained is very general - the only assumptions are that the in-phase and quadrature components (X and Y) are Gaussian random variables. One finds that when the two Gaussian random variables are uncorrelated ($\rho_{xy} = 0$), and $\sigma_x = \sigma_y$, the envelope amplitude distribution reduces to the Rician distribution for non-zero μ_x, μ_y and the Rayleigh distribution when $\mu_x = \mu_y = 0$.

B. Models without Gaussian Assumption

If the in-phase and quadrature components (X and Y) are not Gaussian random variables, there are several models used to characterize the fading channel including Nakagami m -distribution [5] and non-Rayleigh statistics [6].

Nakagami m -distribution is modeled for RF communications channel and is defined as below.

$$p_z(z) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega} \right)^m z^{2m-1} e^{-mz^2/\Omega}, \quad z \geq 0, \quad (7)$$

where Ω is defined as its second moment and the parameter m is defined as the ratio of moments, called the fading figure. The Nakagami distribution contains the Rayleigh distribution as a special case when $m = 1$. It can have fewer deep fades than

the Rayleigh distribution when $1/2 \leq m < 1$, and more deep fades than the Rayleigh distribution when $m > 1$.

For non-Rayleigh fading statistics, K -distribution is one of popular models to characterize reverberant media. The K -distribution is given by [6]

$$p_z(z) = \frac{4}{\sqrt{\alpha}\Gamma(v)} \left(\frac{z}{\sqrt{\alpha}}\right)^v K_{v-1}\left(\frac{2z}{\sqrt{\alpha}}\right), \quad z \geq 0, \quad (8)$$

where v is a shape parameter, α is a scale parameter, K_{v-1} is the modified Bessel function of the second kind, of order $v-1$, and $\Gamma(v)$ is the Gamma function. A special case of the K -distribution, as $v \rightarrow \infty$ and $\alpha v = 2\sigma^2$ remains constant, is a Rayleigh distribution.

Experimental data are analyzed next to identify which model is appropriate for the underwater acoustic communication channel.

IV. FADING MODELS USING EXPERIMENTAL DATA

TREX04 experiment was conducted by the Naval Research Laboratory in April 2004, which took place in the coast of New Jersey. Figure 1 shows a sound speed profile based upon measurement at the site. Acoustic communication data were transmitted from a fixed source to a fixed receiver array at the range of 3.4 km. Water depth in the experimental area is about 70 meters. The source and receivers were located at about 35 meters depth. The vertical array has an aperture of approximately 2 meters, and contains 8 hydrophones with non-uniform spacing. The data presented below are from a single receiver; we observe little difference between the receivers. The data have a high signal-to-noise ratio (SNR) ≥ 30 dB.

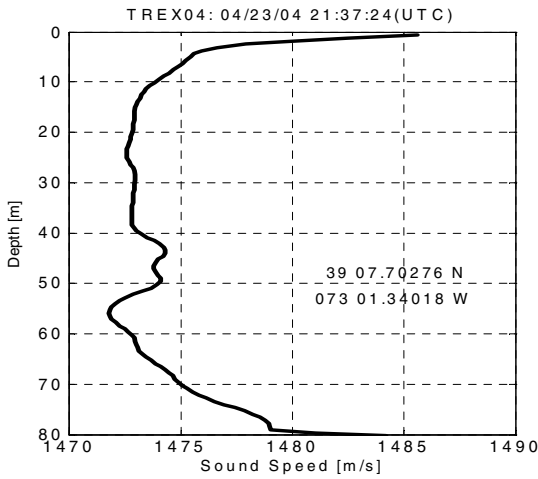


Figure 1. Sound speed profile in TREX04 experiment

An M-sequence signal with a bandwidth of 5 kHz centered at 17 kHz was used to characterize the underwater communication channel. Each transmitted packet lasted approximately 10.7 sec and contained 53 M-sequences. A total of 134 packets, extended over a period of an hour and containing 7102 M-sequences, were analyzed.

The amplitude statistics are plotted in Fig. 2 for different values of f_k . We find that the probability distributions are very similar (within the statistical error) suggesting that the envelope amplitudes at different frequencies (within the band) have independent and identical distributions (iid). [In Fig. 2, three frequencies bins have a slightly different distribution than the rest of the frequencies bins. This difference could be easily caused by a small number of events in the high tail distribution (due to coherent interference between the signal and noise) that would shift the probability distribution to what is shown.

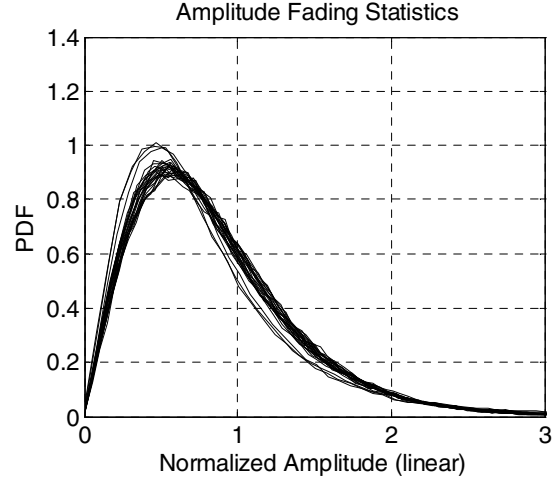


Figure 2. Amplitude fading statistics in all frequency bins

Assuming an iid property, we can include envelope amplitudes of all frequencies (within the band) to obtain more statistical samples. The resulting statistical distribution is fit to the candidate distributions, whose parameters are estimated using the 1st and 2nd moments of the experimental data. The statistics of the experimental data is plotted in Fig. 3(a) (for the 80Hz frequency bin data) and is compared with the Rician/Rayleigh distributions and the distribution using Mikhalevsky's model. (The distribution of the Mikhalevsky's model turns out to be close to the Rayleigh distribution given the measured first and second moments of the data.) We repeat the above analysis using a different signaling design, by varying the frequency bin size Δf from 80Hz to 320Hz, and to 5Hz. The resulting envelope amplitude distributions are plotted in Figs. 3(b) and 3(c) to compare with the modeled probability distributions. These plots show that the models have a poor fit with the data. They suggest that the amplitude statistics for high frequency underwater communication signals are neither Rician nor Rayleigh distribution, nor the more general distribution derived assuming that the in-phase and quadrature components are Gaussian random variables.

From Figs. 3(a)-3(c), one notes that the measurement data do not fit the Nakagami model either, despite the fact that the Nakagami m -distribution can provide more deep fades than a Rayleigh distribution. In contrast, the measurement data seem to fit the K -distribution.

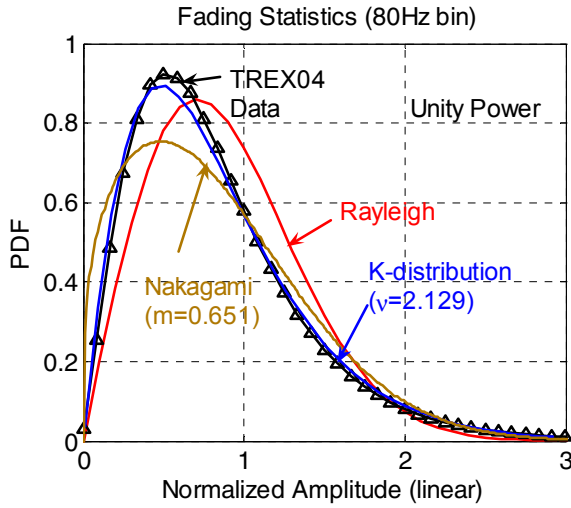


Figure 3(a). Amplitude fading statistics at 80 Hz bin size

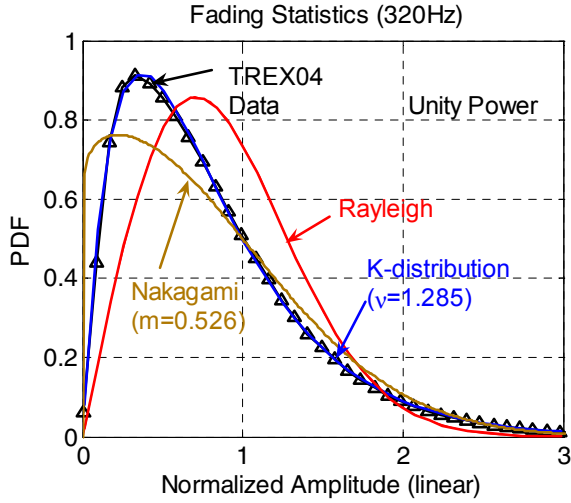


Figure 3(b). Amplitude fading statistics at 320 Hz bin size

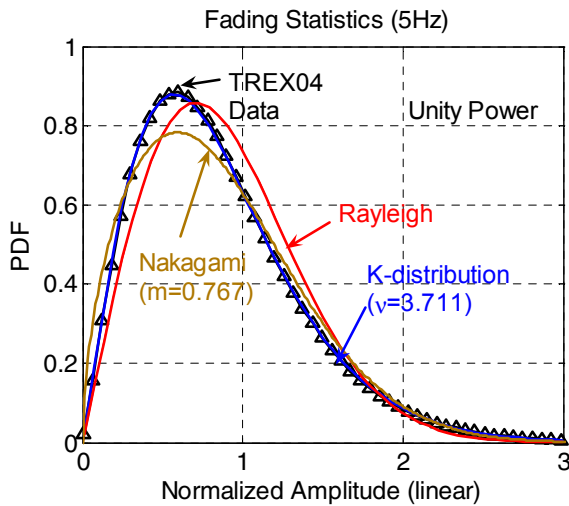


Figure 3(c). Amplitude fading statistics at 5 Hz bin size

The question of interest is what is the underlying mechanism for signal fluctuations (between low and high frequencies) that lead to the Rayleigh/Rician model on the one hand and the K -distribution model on the other hand.

Recall that the Mikhalevsky's model assumes that the random variables X and Y follow stationary Gaussian statistics. This assumption seems to be valid for low frequency signal propagation, but perhaps not appropriate for high frequency signal propagation. High frequency signals may follow quasi-stationary statistics that involve two time scales associated with long-term fading and short-term fading [2]. Over a short time scale, the high frequency signal is heavily influenced by the micro-fine structures (e.g., turbulence) in the ocean. The signal amplitude fluctuation follows a short-term statistics. Over a long time scale, the amplitude fluctuations of the signal will likely be dominated by the fine-structure perturbations of the ocean, assuming that the rapid fluctuations induced by the micro-structures have been averaged out. The signal amplitude fluctuations follow a long-term statistics, which may be different from the short-term statistics. (At low frequencies, the turbulence has no effect on the signal, hence there is only the long term statistics.)

To obtain the long-term statistics, we will introduce an average time scale T . Long-term statistics are obtained by averaging the signal over the time period of T , such that the short-term signal fluctuation has been averaged out. That is, the long-term fluctuation statistics, $|\bar{H}(f_k, T_n)|$, can be

obtained by averaging the signal intensity $|H(f_k, t)|^2$ at a fixed frequency f_k over a period of T . The amplitude, which is the square root of the average intensity, yields a distribution as shown in Fig. 4 for $T = 0.2$ sec. It is well fitted by a log-normal distribution.

The short-term distribution is obtained from individual snapshots. The snapshot data are normalized by the mean amplitude for each period of T , that reflects the long term fluctuations; i.e., removing the effect of long term fluctuations,

$$\tilde{H}(f_k, \tau) |_{T_n} = H(f_k, t) / \bar{H}(f_k, T_n). \quad (9)$$

The normalized data yields a distribution shown in Fig. 5 for $T = 0.2$ sec. One finds that the data are fitted by the Rayleigh distribution.

At the short time scale, the cause of the signal fluctuation is turbulence or other micro-fine structure disturbances. The fluctuation is fully saturated and hence is Rayleigh distribution. At the long time, the signal fluctuation is predominantly due to internal waves or other fine-structure disturbances. The fluctuation is partially saturated and is well described by a lognormal distribution.

At the symbol level, the symbol amplitude envelope statistics follows a joint probability distribution, determined by the short-term probability distribution function conditioned on the amplitude distributions dictated by the long-term probability distribution function. The K -distribution is a mixture of Gamma and Rayleigh distributions. It has been proven that lognormal and Gamma distributions are close approximates of each other [8-9]. Consequently, one finds that

K -distribution is numerically close approximations of a mixture of lognormal and Rayleigh distributions [10].

Next, we evaluate the long-term and short-term statistics based on the “goodness of fit” measure of the root-mean-squared error (RMSE), also known as a fit standard error. The RMSE is defined as below.

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{L} \sum_{i=1}^n (F_i - S_i)^2} \quad (12)$$

where $L=n-m$ indicates the number of independent pieces of information involving the n data points and m parameters of the prospective probability distribution. S_i denotes the sampled statistical function (either the probability distribution or the cumulative probability distribution function) based on data at amplitude x_i , $i = 1, \dots, n$. F_i denotes the sampled statistics at amplitude x_i based on the statistical model. We shall evaluate Eq. (12) for different values of T . For each value of T , we obtain the short-term statistics and long-term statistics from the data. We fit the short-term and long-term envelope fluctuation data with the Rayleigh and long-normal distribution respectively and determine the best parameters that fit the data. Having determined the parameters for the best fit, we then determine the RMSE of the fit using Eq. (12).

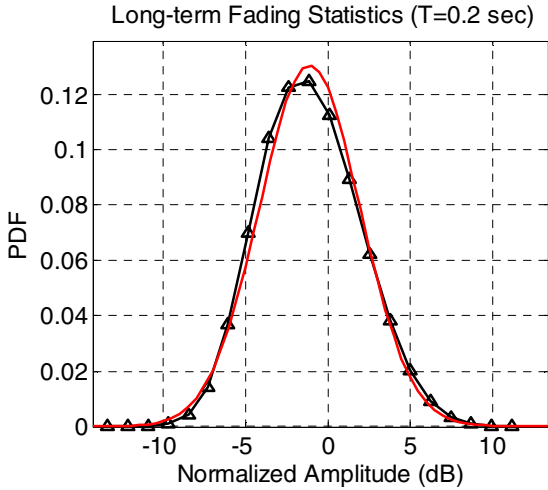


Figure 4. Long-term fading statistics as $T=0.2$ sec vs. lognormal distribution

We study the short-term envelope amplitude statistics by determining the RMSE values of the Rayleigh statistics as a function of the time scale T . The minimum of the test is located at $T \approx 0.3$ sec. The RMSE is $\leq 1\%$ for T between 0.15 and 0.5 sec. It indicates that the Rayleigh distribution is a good fit for the short-term fading statistics during this time window. When T exceeds 0.5 sec, the fit deteriorates significantly. We interpret this result to mean that for $T > 0.5$ sec the effects of fine-structure processes are no longer negligible and need to be included.

For long-term statistical distribution, the RMSE between the data and the Rayleigh distribution is $\leq 1\%$ for T between 0.04 and 0.21 sec, $\leq 1.5\%$ for T between 0.03 and 0.63 sec, and $\leq 2\%$ for a large window of T up to 1.25 sec. This is consistent

with the expectation that the long-term fluctuation should not change significantly with the time window T as long as it is “long-term”. We note that there is no theoretical basis that the distribution has to be Rayleigh. Thus a 2% RMSE is quite reasonable. Figure 6 plots the sum of the short-term and long-term RMSE. The minimum occurs around $T \sim 0.25$ sec. We find that $T = 0.2$ and 0.4 sec yield a reasonable RMSE. For the above data analysis, we use $T=0.2$ sec.

We have also evaluated the fit of the data with the theoretical distribution using the Kolmogorov-Smirnov test statistics [6]. The results are very similar and not explicitly shown here.

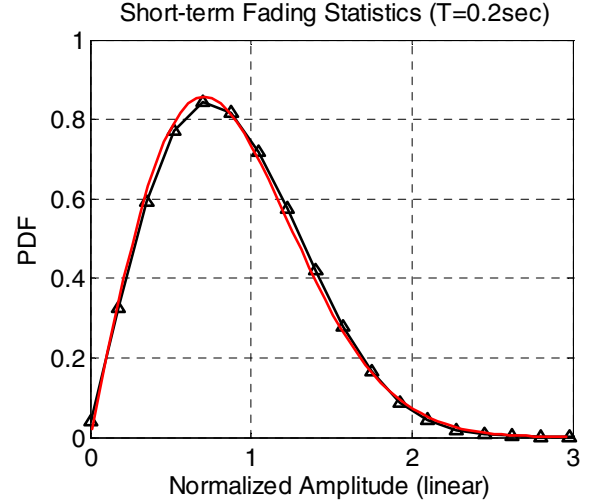


Figure 5. Short-term fading statistics as $T=0.2$ sec vs. Rayleigh distribution

V. CONCLUSIONS

In this paper, we presented envelope amplitude (fading) statistics for narrowband high frequency signals over a wide band of frequencies (15 -19 kHz). The envelope amplitude statistics shows a non-Rayleigh or a non-Rician distribution behavior. The conventional models for the envelope amplitude distributions, developed for low frequency unsaturated, partially saturated and fully saturated signal fluctuations; do not fit the high frequency amplitude statistics data. The reason is that these models assume a fading statistics that is valid for all time scales. Our analysis of the high frequency data indicates two time-scale fading phenomena: long-term versus short-term fading. The division between the two is determined by using RMSE test, which is about 0.2 sec time scale for the TREX04 data.

We found that the long-term amplitude fading statistics follow a lognormal distribution and the short-term amplitude fading statistics follows a Rayleigh distribution. The signal amplitude distribution based on the joint long-term and short-term distributions yields a distribution numerically close to the K -distribution, which is found to be a good fit of the high frequency data.

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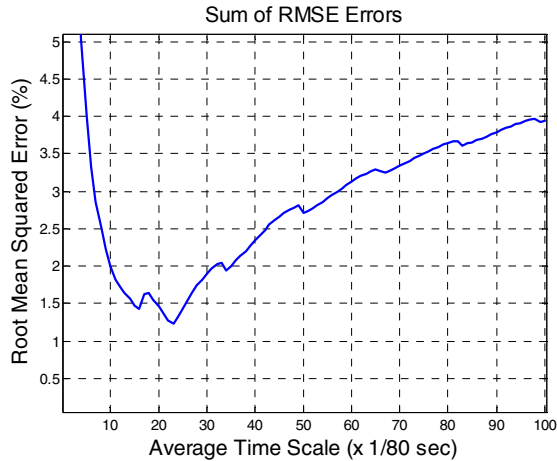


Figure 6. Sum of RMSE errors vs. average time scale

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